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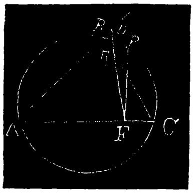
184. Proposed by ERWIN MARTIN, Principal of Schools, Mead, Neb.

If from any point in the circumference of a circle circumscribed about a triangle, perpendiculars are drawn to the sides, or the sides produced, of the inscribed triangle, the lines connecting the feet of the perpendiculars are collinear.

Demonstration by J. R. NITT, Coronal Institute, San Marcos, Tex.; G. I. HOPKINS, A.M., High School, Manchester, N. H.; and G. W. GREENWOOD, B. A., McKendree College, Lebanon, Ill.



Let ABC be a triangle; PD, PE, PF perpendiculars from any point P of circumcircle upon the sides.

To prove D, E, F collinear: Draw BP and CP, DE and DF. The quadrilaterals $BEPD$, $CFPD$ are cyclic.

$\therefore$ $\angle PDE=\angle PBE$, $\angle PDF=\angle PCF$. But $\angle PBE=\angle PCF$, each being measured by $\frac{1}{2}\text{arc}AP$.

$\therefore$ $\angle PDE=\angle PDF$. $\therefore$ D, E, F are collinear.

Also demonstrated by *BEULAH FRAZIER*, Soph., Rolla School of Mines, and *G. B. M. ZERR*.

162. Proposed by J. D. PALMER, Providence, Ky.

Given the distances from the vertices of a triangle, ABC, to the center of the in-circle, to construct the triangle.

Solution by MARCUS BAKER, U. S. Geological Survey, Washington, D. C.

Already two solutions of this problem have appeared in the MONTHLY, an erroneous one in the January number, page 15, and a revised solution in the February number, pages 45-46, yet a word or two more may not be superfluous.

The problem is to *construct* a triangle knowing the distances from the incenter to the vertices.

The correct analysis of the problem, as Zerr shows in his revised solution results in a cubic equation, which means that the problem is insoluble, *i. e.* insoluble in the same sense that the trisection of an angle is impossible. The true answer is that the construction called for can not be made by elementry geometry.

The following is a trigonometrical analysis of the problem.

If A, B, C are the angles of a plane triangle, then

$$2\sin\tfrac{1}{2}A\sin\tfrac{1}{2}B\sin\tfrac{1}{2}C+\sin^2\tfrac{1}{2}A+\sin^2\tfrac{1}{2}B+\sin^2\tfrac{1}{2}C=1\ldots\text{.}(1).$$

This theorem readily results from substituting in the following well known relation

$$\cos\mathrm{A}+\cos B+\cos C-1=4\sin\tfrac{1}{2}A\sin\tfrac{1}{2}B\sin\tfrac{1}{2}C$$

for $\cos A$, $1-2\sin^2\frac{1}{2}A$, etc., and reducing.

Now let α, β, γ be the distances from the incenter to the vertices and r the radius of the inscribed circle; whence

$$\sin\tfrac{1}{2}A=r/\alpha,\quad \sin\tfrac{1}{2}B=r/\beta,\quad \sin\tfrac{1}{2}C=r/\gamma.$$

Substituting in (1), we have

$$\frac{2r^3}{\alpha\beta\gamma}+\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)r^2=1,\text{ or }r^3+\tfrac{1}{2}\left(\frac{\alpha\beta}{\delta}+\frac{\beta\gamma}{\alpha}+\frac{\gamma\alpha}{\beta}\right)r^2=\tfrac{1}{2}\alpha\beta\gamma.$$

Example. Let $\alpha=275$, $\beta=325$, $\gamma=429$; then $r^3+539\frac{1}{6}r^2-19170937\frac{1}{2}=0$; whence $(r-165)(r^2+704\frac{1}{6}r+116187\frac{1}{2})=0$.

$\therefore r=165$. Also, $r=-137.97+$ and $r=842.13+$.

CALCULUS.

140. Proposed by **C. C. BEBOUT, Professor of Mathematics, Elgin High School, Elgin, Ill.**

A pole two inches in diameter is set vertically in a level plat of ground. At a point ten feet from the ground a string is attached. A man holds the other end of the string and walks about the pole keeping the string stretched taut, and his hand at a constant distance of four feet from the ground, till the string is all wound upon the pole. If string is ten feet long, how far has his hand moved in the operation?

Solution by **G. B. M. ZERR, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.**

Let n be the number of times the string 10 feet feet in length will encircle the cylinder in a height of 6 feet. Circumference of cylinder$=\frac{1}{6}\pi$ feet, x=distance between successive portions of the string on the same generating line.

Then $\sqrt{[(\frac{1}{6}\pi)^2+x^2]}=10/n$ also $nx=6$.

$$\therefore n\sqrt{\left(\frac{\pi}{6}\right)^2+\left(\frac{6}{n}\right)^2}=10\text{ or }\frac{\pi^2n^2}{36}=64.\quad\therefore n=\frac{48}{\pi},\ x=\tfrac{1}{8}\pi.$$

The distance moved by the hand is given on page 319, No. 9, Vol. I, of the MONTHLY, and is there worked out in full. It is $s=2\pi^2r^2m^2$ where r is the radius of the cylinder and m is the number of times the string is wound around the cylinder or the same as the value of n above. $r=\frac{1}{12}$ feet, $m=n=48/\pi$.

$\therefore s=32$ feet.

Also solved by *T. T. DAVIS.*

141. Proposed by **B. F. FINKEL, A.M.. M.Sc.. Professor of Mathematics and Physics, Drury College,Springfield, Mo.**

The curve $r^n=a^n\sin n\theta$ rolls along a straight line. Show that the intrinsic equation to the evolute of the locus of the pole is $s^n=a^m[1+1/n]^n\sin\psi$. [Edward's *Differential Calculas*, page 502.

Solution by **WILLIAM HOOVER, A. M.. Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.**

The distance from the point of tangency to the pole is normal to the path of the pole. Let this distance, which is a radius vector of the rolling curve, be r, and the perpendicular from the pole upon the fixed line be p.

Taking the fixed line as the axis of x, p is an ordinate of the pole, and r is the length of normal and is expressed by